Emittance Compensation Overview

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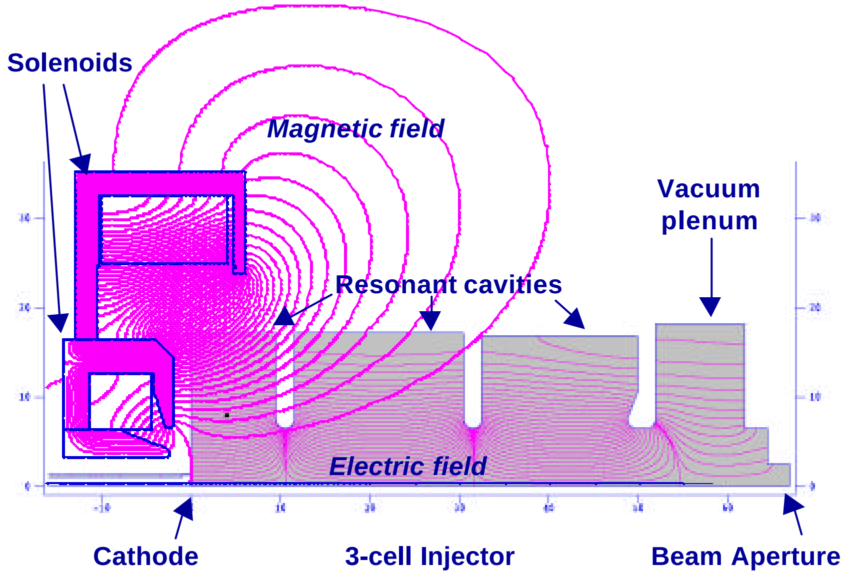
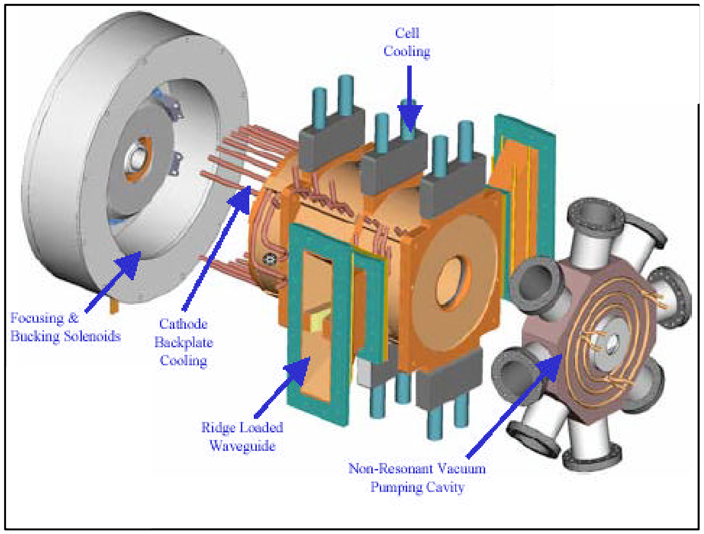
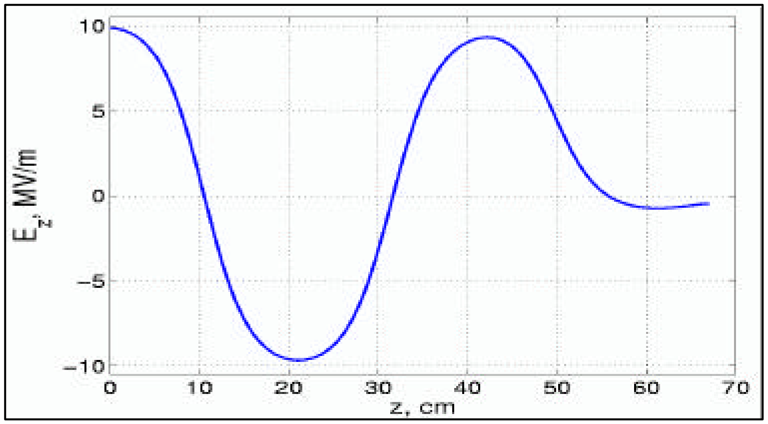
Outline

- Mechanism of transverse plasma oscillations and emittance oscillations
- Comparison to induction linacs
- Dominant mechanisms for final emittance
    - Landau damping of emittance oscillations
    - Wavebreaking

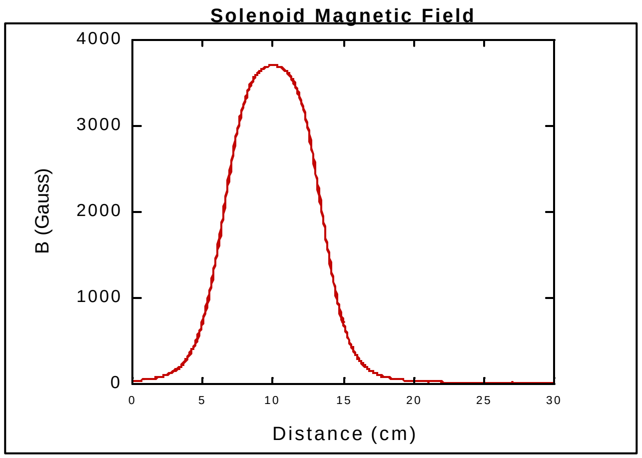
Photoinjector Schematic





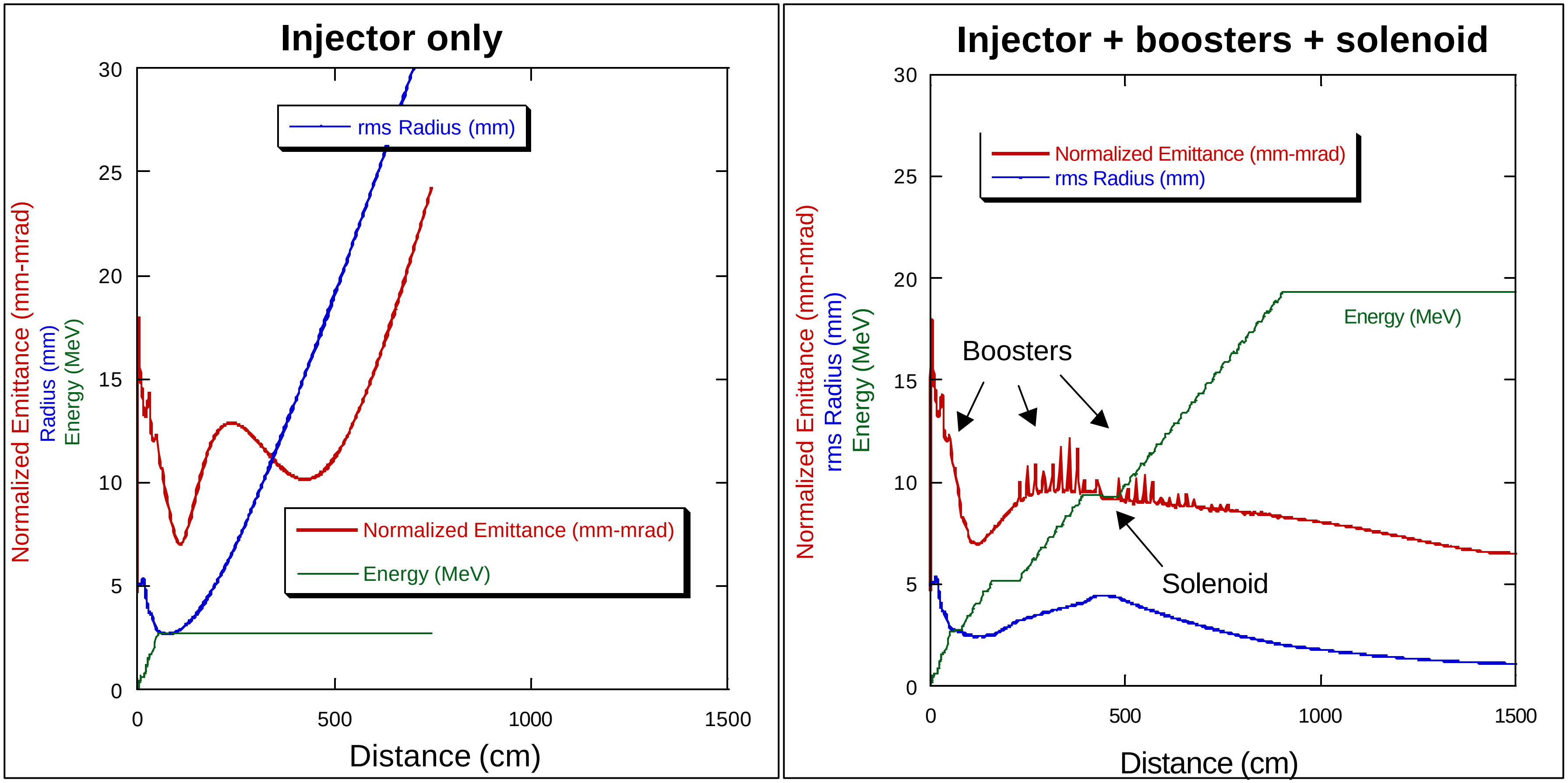


On-axis RF Electric Field



On-axis DC Magnetic Field

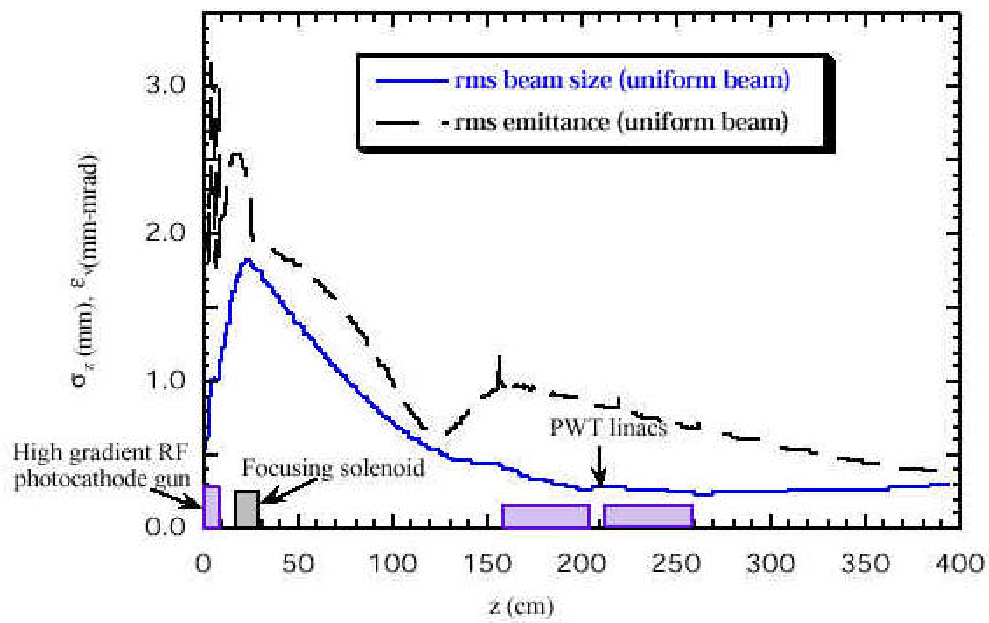
Emittance Oscillations in Nominal RF Gun Design (700 MHz, 3 nC, 9 ps)



Adding booster linacs and a second solenoid allows different phase-space envelopes to realign at higher energy, resulting in a lower beam emittance

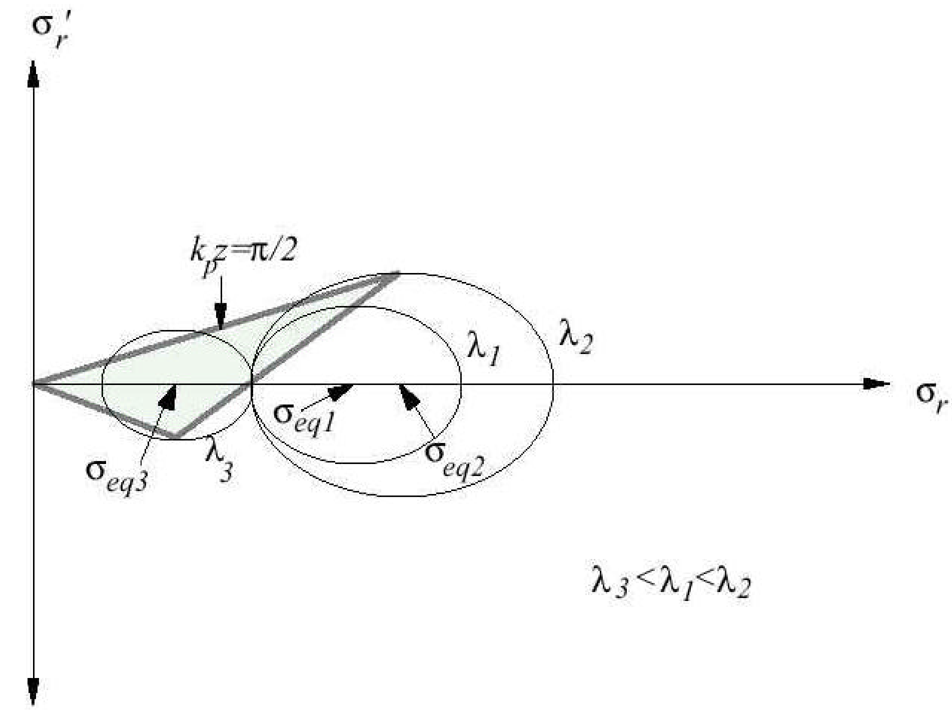
Plasma Oscillations Dominate Emittance Evolution

Major features common for many rf gun designs:



Typical emittance evolution profile for rf linac with photocathode gun (UCLA design), emittance minimum at full plasma period (S. G. Anderson and J. B. Rosenzweig, Phys. Rev. ST Accel. Beams **3**, 094201 (2000))

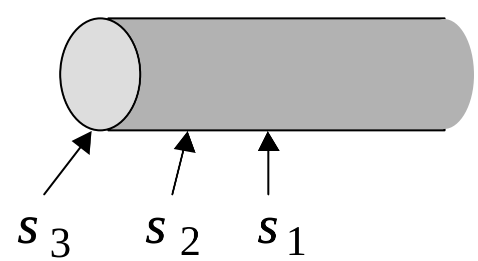
Emittance Oscillations Tied To Plasma Oscillations



$$s'' + Ks - K_s / s = 0$$

$$K_s = 2I(r) / I_A g^3 b^3$$

$$s_{eq} = \sqrt{K_s / K}$$



Bow-tie phase space distribution forms during the plasma oscillations of particles with different equilibrium radii

Plasma Oscillations With Acceleration

We get the same conclusion about transverse plasma oscillations with axial acceleration, although the math is more complicated:

Radial eqn of motion: $$s'' + s'\frac{g'}{g} + sK\left(\frac{g'}{g\sin f}\right)^2 - \frac{K_s}{sg^3} = 0$$

$$y = \ln g \qquad S(z) = K_s(z)/g'^2$$

Cauchy transform (Serafini and Rosenzweig)

$$\frac{d^2 s}{dy^2} + \Omega^2 s = \frac{Se^{-y}}{s}$$

Transformed eqn of motion

Equilibrium radius $$s_{eq} = \sqrt{\frac{S}{\frac{1}{4}+\Omega^2}}\, e^{-y/2}$$

The solution of a trajectory perturbed from the equilibrium trajectory by an amount d_0 is given below. The phase of these oscillations is clearly more complicated (bunch slices start out of phase), and one can achieve better alignment of the slices if there is some flexibility of the initial conditions at the cathode (some initial divergence or rf focusing).

$$s = \sqrt{\frac{S/g}{\frac{1}{4}+\Omega^2}} + d_0 \cos(w \ln g + q)$$

$$s' = -\frac{g'}{2}\sqrt{\frac{S/g^3}{\frac{1}{4}+\Omega^2}} - d_0 \frac{wg'}{g}\sin(w \ln g + q)$$

DARHT Is The Next-Generation U.S. Facility For Radiographic Hydrodynamics Testing

Aerial view of DARHT

A “hydrotest” on the operational first axis of DARHT

- **Flash radiography measurements require 3 essential capabilities :**

  1. ***high-resolution***
  2. ***multiple-views (3D reconstruction)***
  3. ***multiple times (dynamic code benchmarking)***

- **DARHT is the first U.S. facility that begins to provide these capabilities :**

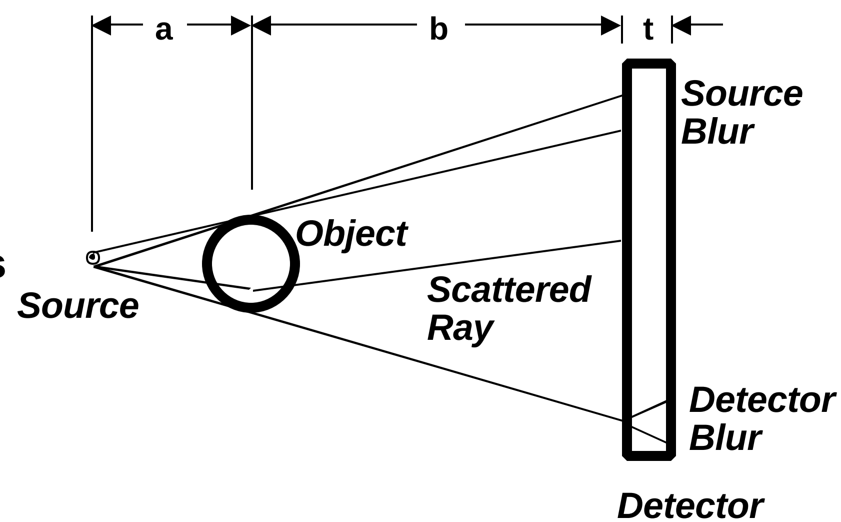
  1. ***high-resolution (0.2-mm - 0.5-mm rms edge location)***
  2. ***multiple-views (2-axes, can be simultaneously viewed)***
  3. ***multiple-times (single-pulse 1st axis, 4-pulse in 2-μsec 2nd axis)***

Some DARHT Accelerator Features

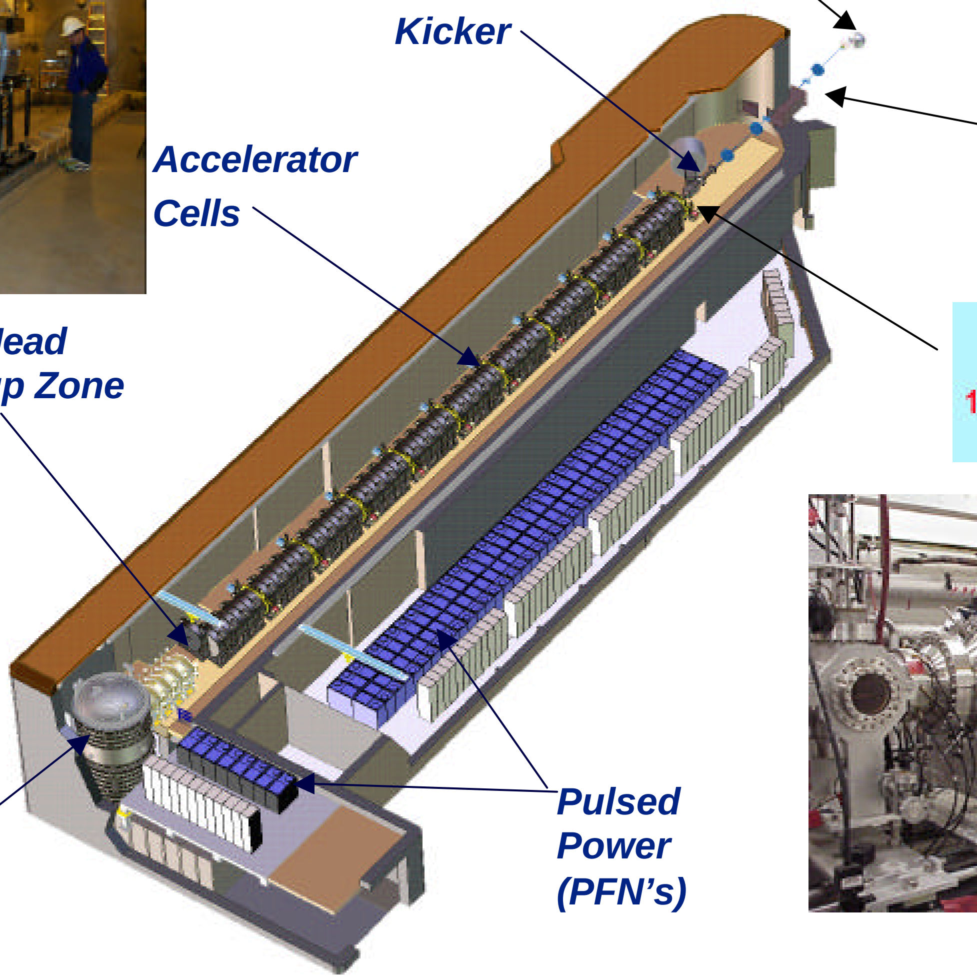
- 2-kA, 20-MeV accelerator
- Very low gradient (about 1/2 MeV/m)
- Very high current leads to large potential depression in beam (150 keV)
- Emittance is dominant parameter for radiography - reducing emittance will give better resolution

Parameter	*Requirement*
Norm. Emittance (4-rms, on target, π mm-mrad)	**1500**
Energy spread	**± 1%**
Micropulses on target (number and pulse-width)	**4, each > 20-ns**
X-ray spot size (mm, 50% MTF)	**2.1**
X-ray dose (R @ 1-m)	**100-500**

Radiographic magnification = (b/a) + 1

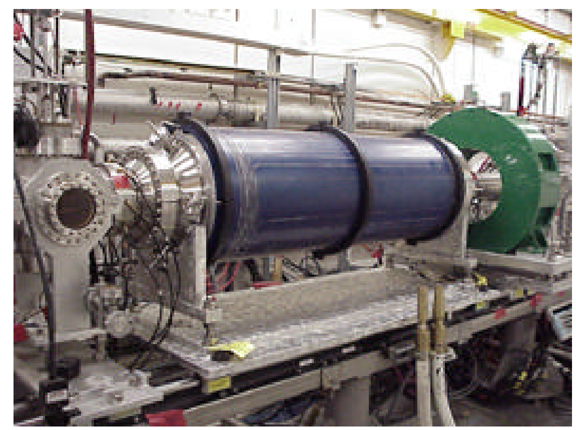


DARHT Phase 2 Accelerator Layout



18.4 MeV
2 kA
1500 mm-mr

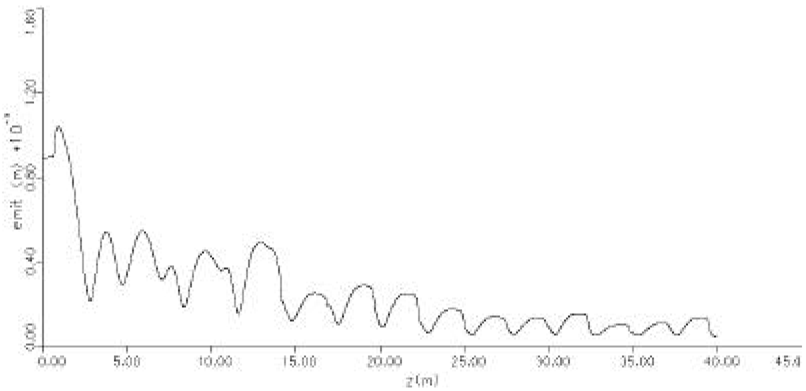
18.4 MeV, 2 kA, 2μs
1000 mm-mr

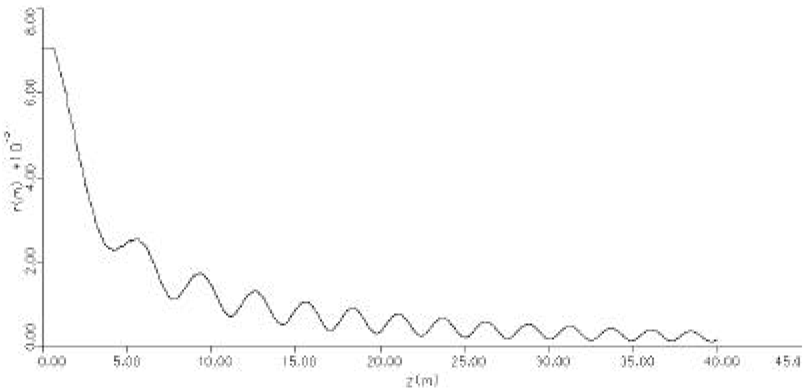


Kicker

Emittance Oscillations In DARHT Lead To Additional Insights

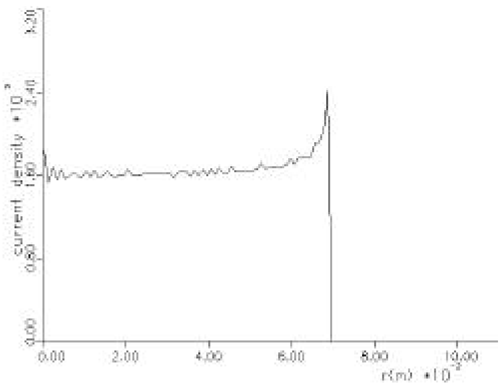
Control of emittance oscillations important for high-brightness induction linacs like DARHT:

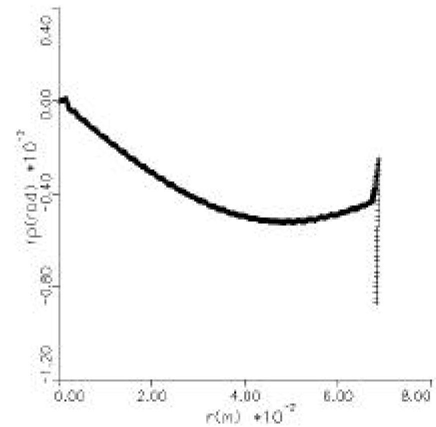




Three key physics issues - (1) diode nonlinearities initiate emittance oscillations (which are out of phase), (2) nonlinear forces due to the potential depression of the beam curve the phase space and *lead to the emittance decreasing*, (3) wave breaking can lead to uncontrollable thermalization and breakdown of the emittance oscillations

Solenoid Field, Anode Hole, and Focus Electrode All Contribute To Both Nonlinear Phase Space And Nonuniform Beam Density





The nonlinear density and curvature result in equivalent out-of-phase plasma oscillations, at different radial positions - this requires a specific radial focusing rate to work out the phase mismatch

Also see beginnings of wavebreaking in phase space

Nonlinear Effect From Potential Depression Changes The Plasma Period

Even if space-charge force is linear, effect of potential depression leads to nonlinear effect:

$$m\frac{d(g\dot{r})}{dt} = eE_r + e(v_q B_{dia} - v_z B_q) + ev_q B_{ext} + \frac{gmv_q^2}{r}$$

Using $$\frac{d}{dt}g\dot{r} = \dot{r}\frac{dg}{dt} + g\ddot{r} = \frac{eE_r}{mc^2}\dot{r}^2 + \frac{eE_z}{mc^2}\dot{z}\dot{r} + g\ddot{r}$$

We end up with

$$r''mv_a^2 = \frac{eE_r}{g^3} + \frac{1}{g}\left(ev_q\left(B_{ext} + B_{dia}\right) + \frac{gmv_q^2}{r}\right) - \frac{e}{c^2 g}v_a^2 E_r r'^2 - \frac{e}{c^2 g}v_a^2 E_z r'$$

Rewriting:

$$r'' = \left[\frac{eE_r}{m v_a^2 g_a^3} - \frac{r}{lf} - \frac{eE_r r'^2}{g_a m c^2} - \frac{eE_z r'}{g_a m c^2} \right]$$

The E_r term is cubic in radius, and can be used to work the curvature out of the phase-space during the emittance oscillations *without* an emittance increase even if the space-charge force is linear

Electron diodes typically give initially nonlinear densities - this is good, because the nonlinear focusing effect will lead to an emittance growth if the density is initially uniform

Multiple Emittance Oscillations Make Us Think About Thermalization Mechanisms

1. Coulomb Collisions - thermalization time way too long to be of interest in practical rf or induction linacs

2. Landau Damping - spread in transverse oscillation periods, long but induction linacs approach it (shouldn't be confused with the initial spread of oscillation phases)

3. Wavebreaking - dominate thermalization effect for both rf and induction linacs (Anderson)

Thermalization From Landau Damping Takes A Long Time

The transverse motion of a particle in a slice of the beam in a uniform focusing channel of normalized strength *K* is given by (nonaccelerating)

$$s'' + Ks - \frac{\hat{K}_s}{s} = 0$$

The equilibrium particle radius (no acceleration) is given by

$$s_{eq} = \sqrt{\frac{\hat{K}_s}{K}}$$

We write the particle radius as $s = d + s_{eq}$

If the beam is rms matched and the density nonuniformity is small, we get this equation:

$$d'' + \left(2K + K\left(\frac{d}{s_{eq}}\right)^2\right)d - K\left(\frac{d}{s_{eq}}\right)^2 s_{eq} = 0$$

For small displacements, this equation indicates emittance oscillations: $d'' + (2K)d = 0$

Oscillation frequency independent of local beam space-charge force

Distance for plasma oscillation: $z_{plasma} = 2p/(2K)^{1/2} = pr_e \sqrt{\frac{I}{I_A} g^3 b^3}$

For next order displacements, nonlinear equation has solution: $d = d_0 + d_1 \cos \Omega z$

Where

$$\Omega = K^{1/2}\left(2 + \left(\frac{d_{rms}}{s_{eq}}\right)^2\right)^{1/2} \qquad d_0 = \frac{K\left(\frac{d_{rms}}{s_{eq}}\right)^2}{2K + K\left(\frac{d_{rms}}{s_{eq}}\right)^2} s_{eq}$$

We can rewrite this in terms of the rms current density and the average current density seen by a particle:

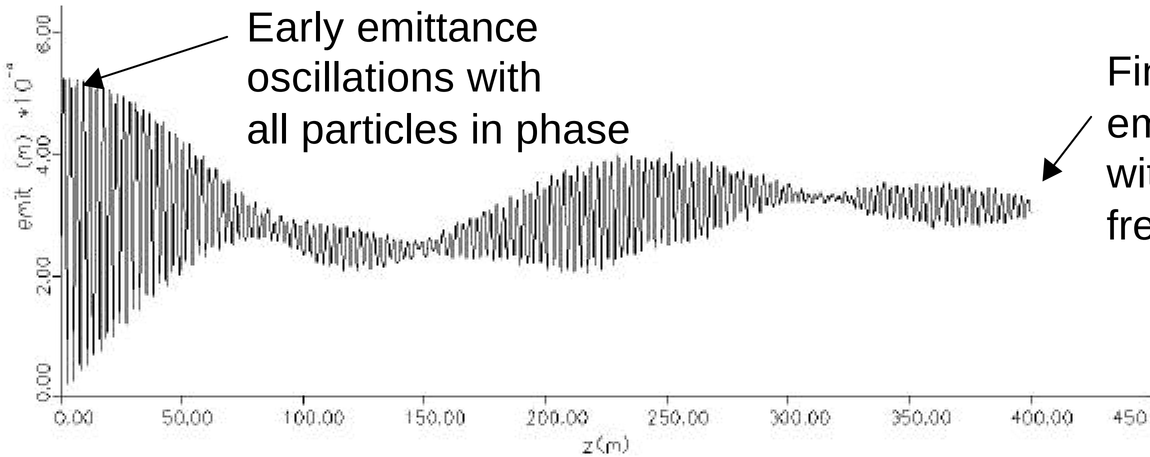
$$\Omega(r) = K^{1/2}\left(2 + \frac{1}{2}\left(\sqrt{\frac{J_{rms}}{J}} - 1\right)^2\right)$$

So particle oscillations stay in phase for small axial distances, but gradually go out of phase; 90 degree spread is generated (entropic thermalization) when

$$\frac{z_{Landau}}{z_{plasma}} = \frac{1}{\left(\sqrt{\frac{J_{rms}}{J}} - 1\right)^2}$$

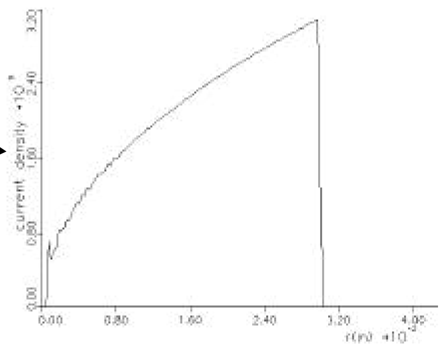
Nominal simulation - 4 kA, 4 MeV showing emittance oscillations:

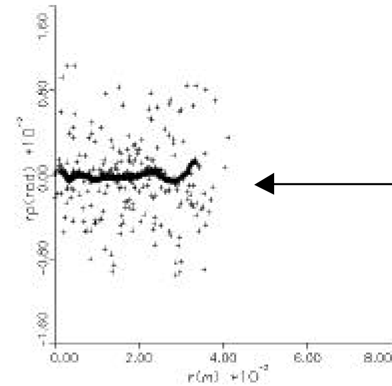
Early emittance oscillations with all particles in phase



Final thermalized emittance - agrees with nonlinear free energy

Initial radial current distribution





Final phase space distribution - kinks approximate final ellipse after long thermalizational

Landau thermalization takes about 35 plasma periods (nonlinear equation estimate indicates 30 periods)

Wave Breaking Leads To Final Emittance For Both RF and Induction Linacs

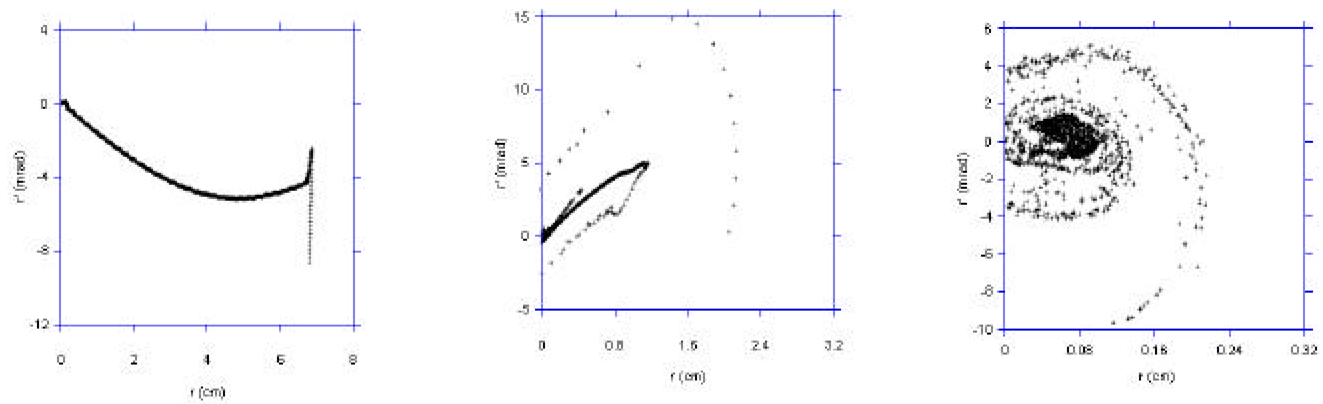
At this point we know that emittance oscillations will occur if the beam either has an initial emittance or some initial density profile mismatch. If no wave breaking occurs, these oscillations will persist for very long times and distances before thermalization. However, if the beam wave breaks, thermalization will occur much faster – essentially at the rate that the particles wave break. Note that the characteristic time scale for wave breaking is a radial compression, or about a ¼ betatron period. This time scale has been empirically described before (Reiser). Although we can not make general statements about the rate of wave breaking for particles in transport, we can easily estimate a characteristic emittance which will always lead to significant wave breaking in the first betatron period. Wave breaking of about ½ the particles will occur for a uniform density beam with an initial emittance exceeding

$$e_{w-b} = 4r_e\sqrt{(I/I_A)/gb}$$

High-current, high-brightness electron beams are under this limit, and do not undergo immediate, large-scale wave breaking. However, rapid phase-space mixing is common for ion beams (this emittance threshold is only 0.2 mm mrad for a 1-GeV, 100-mA, 1-mm radius hydrogen beam). This consideration explains why high-brightness electron beams do not thermalize like ion beams.

Wave Breaking Leads To Final Emittance

Wave breaking dominates final DARHT emittance:

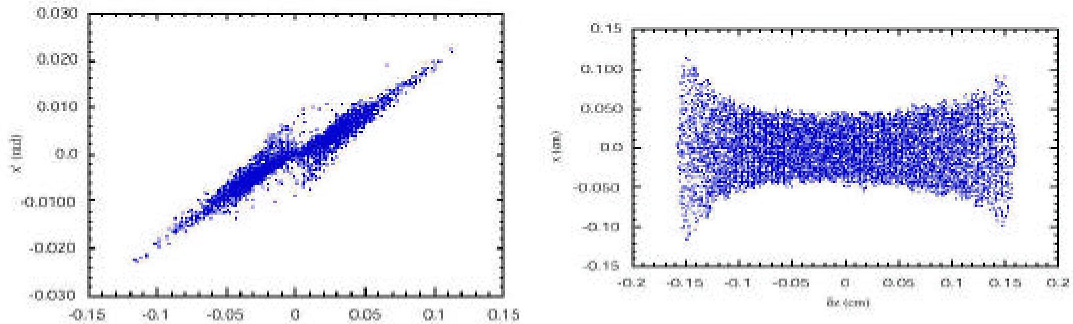


Wave breaking particles passing through the beam core forming a halo early on, and final evolved distribution consists of wave breaking particles spiraling about the 2:1 resonance

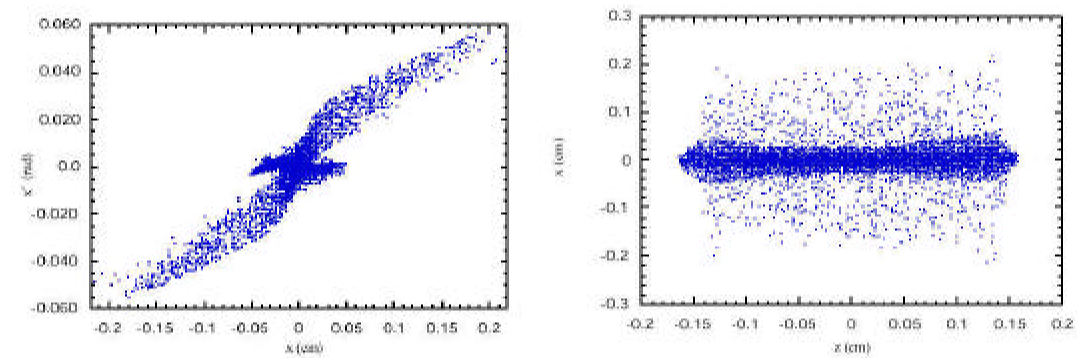
Wave Breaking Also Dominates Emittance In RF Linacs

Large-scale wavebreaking can occur for radially nonuniform distributions:

Uniform case - bowtie distribution results from spread in plasma frequencies



Nonuniform case - horizontal component from wavebreaking - final emittance is 4 times larger



Summary

1. Thermalization (binary collisions and Landau damping) sufficiently long so high-brightness electron beams *in both rf and induction linacs* undergo 1 or more coherent emittance oscillations

2. Proper conditions (cathode curvature or predictable nonlinear forces) can lead to good alignment of phase-space ellipses

3. Wavebreaking tends to dominates final emittance after emittance oscillations *in both rf and induction linacs*

4. Analytic model for wavebreaking would be very useful for estimating residual emittance after compensaion